Effect of size on a simple model of ferromagnetism

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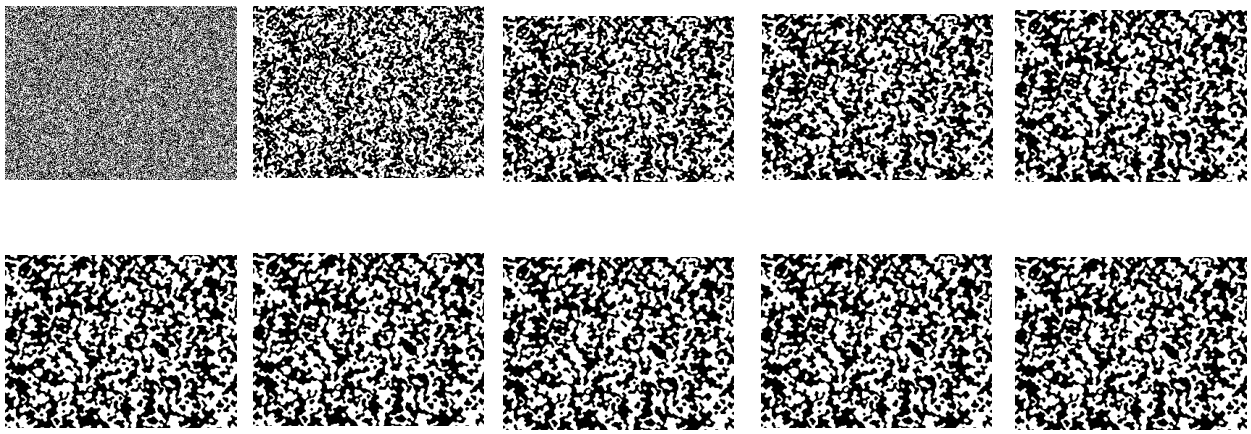
Abstract

A simplified Ising model was used to model ferromagnetism in the abscence of temperature effects and external magnetic fields. An array of random boolean values was used to simulate an initial condition in which atoms have an arbitrary spin up or down. Coupling forces are simulated by applying rules which determine each cell's new state, and the array is updated iteratively until it reaches a stable state. Two measurements, magnetization and domain size, are evaluated over the course of each iteration and in relation to the array size.



Introduction and Theory

Magnet dipoles normally align in opposite directions. But in materials that are ferromagnetic, many small dipoles will naturally align to form magnetic domains. Although the domains themselves are lined up oppositely in accordance with classical physics, the ferromagnetic effect of aligned elements cannot be explained strictly according to a classical view of electromagnetism. Quantum theory can explain the strange clumping of aligned dipoles. This effect can be approximated using the Ising model. This model was conceived by Ernst Ising based on one proposed by Wilhelm Lenz. The model is useful for the study of many physical and social phenomenon, as it simply models the behaviour of a single element attempting to match the ones that surround it.

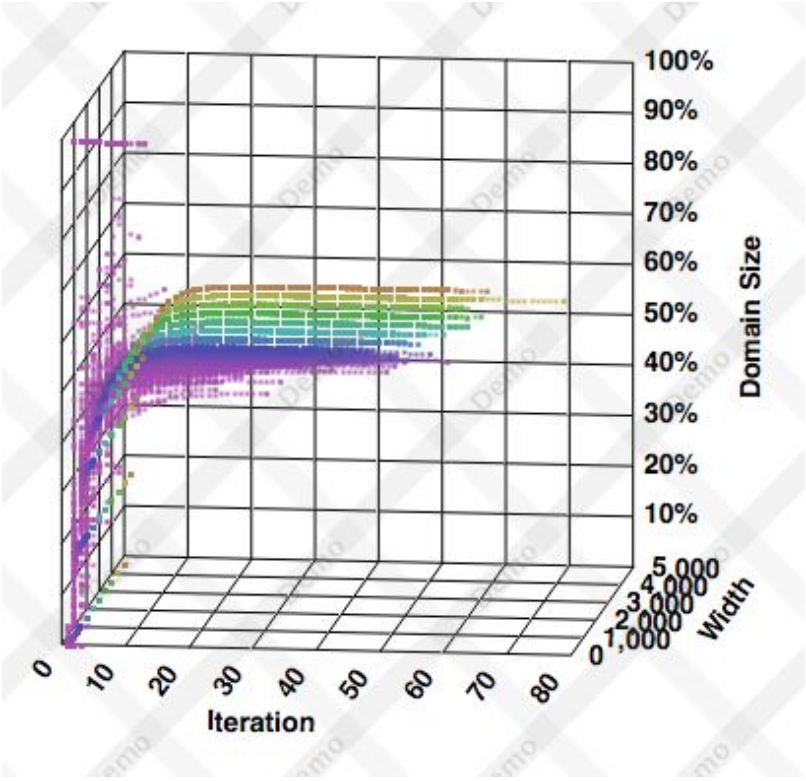
Experimental Method

This experiment used a simplified version of the Ising model. A square matrix represented the ferromagnetic system. Each element of the matrix was set to a one or zero arbitrarily, so that each cell came to represent a dipole with an initially random spin, either up or down. According to the Ising model, each element will switch states, from a one to a zero or vice versa, to match a majority of the elements around itself. For example, if an element had the value 'one', and the eight adjacent elements were divided evenly into 4 ones and 4 zeros, the center element would not flip because there is no majority. The switching tallies are performed iteratively, each step creating a new matrix based on the previous one. Consider a step in which a 'one' cell is surrounded by five 'zero' cells on top, and three 'one' cells below. The corresponding center cell in the next step would be flipped, but its new state would not have a flipping effect on the lower cells of the original step. The continuously randomizing effect of temperature on the ferromagnetic system, a key part of the original model, was disregarded.

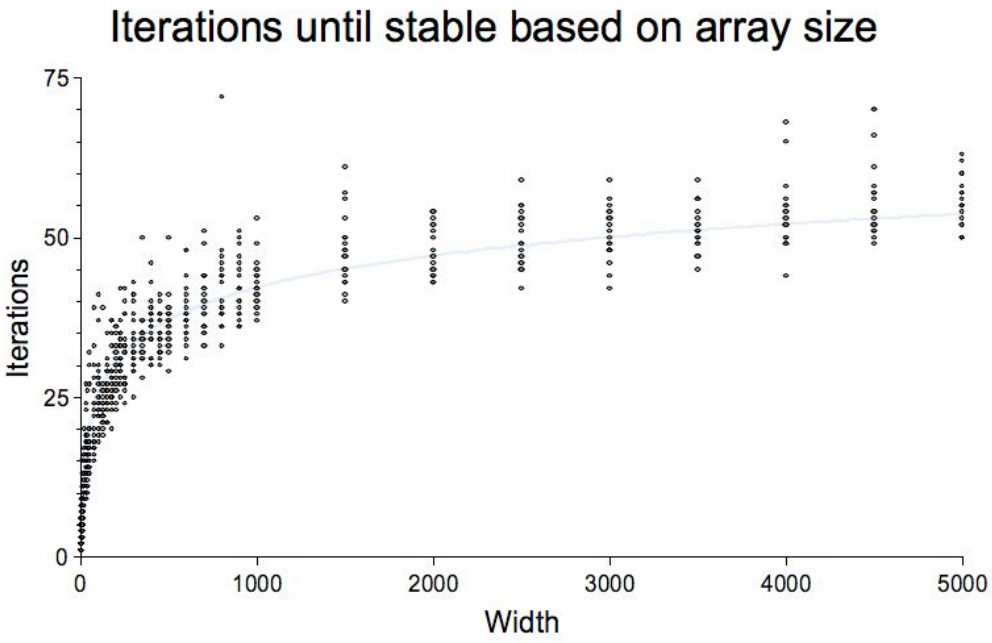
The implementation started out as a demonstration in a spreadsheet in a popular office suite, which provided a way to grasp the concept of the model. However, the spreadsheet did not scale well in both speed and ease of use for larger matrixes. The simplified model was re-implemented on two other platforms, one which made it easy to create visual depictions of the model as it progressed, and one which allowed for faster calculation of large grids. See the Conclusions for information on source code and sample results not given in this report.

Results and Analysis

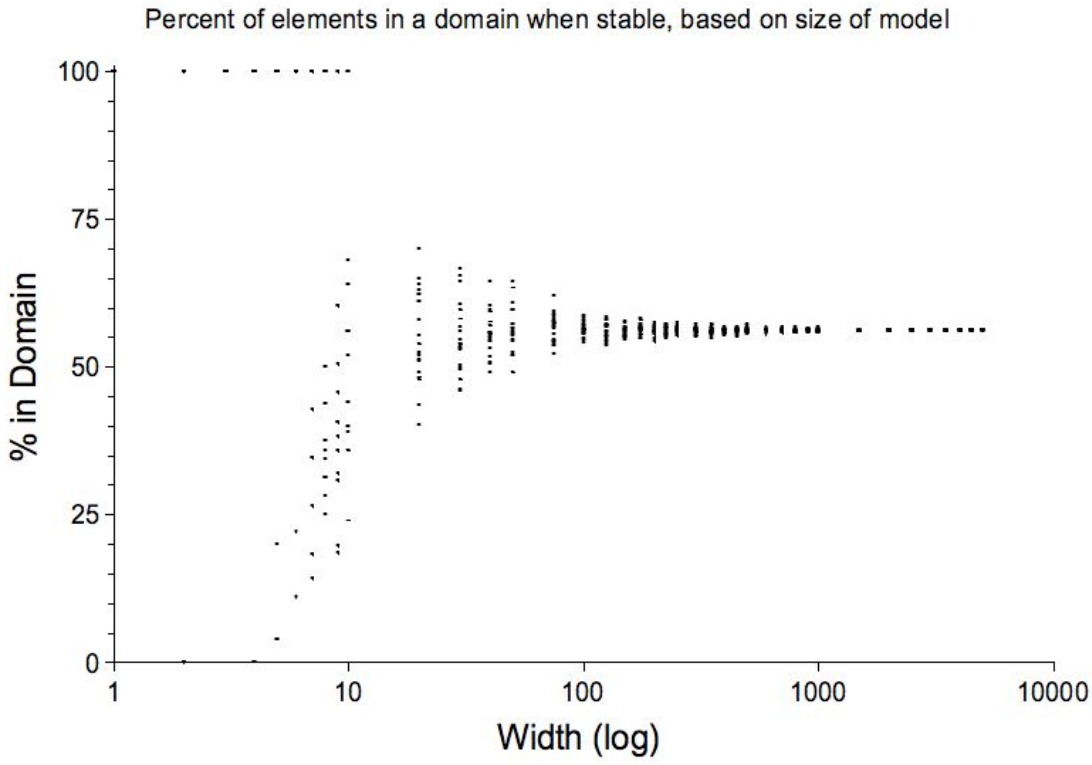
As you can see from this massacred three-dimensional plot, dipoles quickly aligned themselves into domains, yet it took most larger grids a sizable amount of iterations to become fully stable:



The total number of iterations seemed to taper off logarithmically as the size of the simulated material increased, although there was wide variance between each of twenty runs for the sizes calculated [some data points overlap :-(]



After becoming stable, the percentage of elements completely surrounded by those of like spin was calculated. For grid sizes of about 100x100 or larger, the final 'domain size' of the material becomes remarkably constrained:



Conclusions

The data show that size has a definite effect on this simple Ising model. The very small models come to stability rather quickly, but have extreme variations in the final domain size. The large models take longer to stabilize (although the number of iterations is graciously logarithmically proportional) and all end up with about 56% of their dipoles inside of a domain.

For source code and sample data, refer http://homepages.dordt.edu/~nthnlvnd/soft/

Acknowledgments

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